Total No. of Printed Pages:3

SUBJECT CODE NO:- P-273
FACULTY OF ENGINEERING AND TECHNOLOGY
S.E. (All Branches) Examination MAY/JUNE-2016
Engineering Mathematics -IV
(Revised)

[Time: Three Hours] **[Max Marks:80]**

"Please check whether you have got the right question paper."

N.B
1) Question numbers 1 and 6 are compulsory.
2) Solve any two questions, from remaining of each section.
3) Figures to the right indicate full marks.
4) Assume suitable data, if necessary.

Section A

Q.1 Solve any five:- 10

a) Verity the Cauchy's Riemann equations for the function $f(z) = e^{-\bar{z}}$
b) Find the harmonic conjugate of u=2xy+3y.
c) Evaluate $\int_0^{1+i} e^{|z|^2} Re(z)dz$ along the straight line z=0 to z=1+i
d) Evaluate $\int (x + y)dx + ix^2y\,dy \;\; along \;\; y = x^2$ from (0,0) to (3,9).
e) Find the residue of $f(z) = \frac{1}{z^2.e^z}$ at each pole.
f) Find the image of the line y=0 under the transformation w=log z.
g) Solve $\frac{\partial^2 u}{\partial y^2} = 0, \;\; where\; u(x, 0) = x^2, u(x, 1) = 1$

OR

find the z-transform of

$$u(k) = 1, K \geq 0$$
$$= 0, k < 0$$

h) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y}$,where u(x,0)=$6e^{-3x}$

OR

Find the z-transform of Ka^K, $K \geq 0$.

Q.2

a) If $f(z) = u + i\vartheta$ is analytic then find f(z), where $u + \vartheta = \frac{1}{\gamma}[(r^2 + 1)cos\theta + (r^2 - 1)sin\theta]$ 05
b) Evaluate$\oint_C \frac{(Z-3)}{(Z^2+2Z+5)} dZ$, where C is $|Z + 1 - i| = 2$ by using cauchy's integral formula. 05
c) Solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, subject to the condition y(0,t)=y(l,t)=0 and $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0, y(x, 0) = sin\frac{\pi x}{l}$ 05

OR

Find the Z-transform of $sin^2\frac{K\pi}{4}$

Q.3

a) Find the harmonic conjugate of $u = e^x\, cosy + x^3 - 3xy^2$, also find corresponding analytic function f(z). 05
b) Evaluate c$\int \bar{z}^2 dz, where\; C\; is\; |z - 1| = 1$ 05
c) Solve $\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, where 0<x<5 for $\frac{\partial u}{\partial x}(0, t) = 0, \frac{\partial u}{\partial x}(5, t) = 0\; and\; u(x, 0) = x$ 05

OR

Find the inverse z-transform of $\frac{Z^2}{Z^2+4}$

Q.4 a) Find the image of the triangular region bounded by the lines x=0, y=0 and x+y=1 under the transformation w=2iz. 05

b) Evaluate $\oint_c \frac{(2Z+1)}{Z^2-Z-2}dz$, where C is $|Z| = \frac{3}{2}$ by using Cauchy residue theorem. 05

c) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the condition $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$, $\left(\frac{\partial u}{\partial x}\right)_{x=a} = 0$, $\left(\frac{\partial u}{\partial x}\right)_{x=b} = 0$ and $u(x,a) = u_0 \cos\left(\frac{\pi x}{a}\right)$. 05

OR

Solve – by z- transform $y_{n+2} - 5y_{n+1} + 6y_n = 5^n, if\ y_0 = 0, y_1 = 0.$

Q.5 a) Find the bilinear transformation which maps the points z=2,i,-2 on to the points w=1, i,-1. 05

b) Expand $f(z) = \frac{7Z-2}{(z+1)z(z-2)}\ for\ 1 < |z+1| < 3.$ 05

c) Evaluate $\int_0^{\pi} \frac{d\theta}{17-8\cos\theta}$ by residue theorem. 05

Section B

Q.6 Solve <u>any five</u> 10

a) Find the Laplace transform of $\frac{Sin\omega t}{t}$

b) Find the Laplace transform of $e^{4t}t^{3/2}$

c) Find the Laplace transform of $te^t f(t)$

d) Find the inverse Laplace transform of $\frac{1}{s}\left(\frac{s-3}{s+3}\right)$

e) Find the inverse Laplace transform of $\frac{1}{S^3+4s}$

f) Find the inverse Laplace transform of $\frac{e^{-\pi s}}{s^2+2s+2}$.

g) Find the Fourier sine transform of $\frac{1}{x}$

h) Find the Fourier transform of

$$f(x) = 0, if\ x \le a$$
$$= 1, if\ a < x < b$$
$$= 0, if\ x \ge b$$

Q.7 a) Evaluate $\int_0^{\infty} \frac{sinat\ sinbt}{t} dt$ 05

b) Find inverse Laplace transform of $\frac{1}{s}cot^{-1}s$ 05

c) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, x>0, t>0, subject to the conditions 05

1) U=0, when x=0, t>0
2) U=1, 0<x<1
   $= 0, x \ge 1$
3) U(x,y) is bounded.

Q.8 a) Find the Laplace transform of $\int_0^t t\ sinh2tdt$ 05

b) Find the inverse Laplace transform of $\frac{1}{(S-2)(S+2)^2}$ by convolution theorem. 05

c) Find the Fourier cosine transform of $e^{-ax}\ sinax$. 05

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Q.9 a) Find the Laplace transform of periodic function $f(t) = e^{at}\ for\ 0 < t < 2\pi.$ 05

b) Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 6t - 8, y(0) = 0, y'(0) = 0$ by Laplace transform method. 05

c) Solve the integral equation $\int_0^\infty f(x) \sin \lambda x\ dx = \frac{sin\lambda}{\lambda}$ 05

Q.10 a) Express the following function in terms of Heaviside unit step function and hence find their Laplace transform 05

$$f(t) = sin2t, 2\pi < t < 4\pi$$
$$= 0, t > 4\pi$$

b) Solve $\frac{dy}{dt} + 2x = sin2t, \frac{dx}{dt} - 2y = cos2t, x(0) = 1, y(0) = 0$, by Laplace transform method. 05

c) Find the Fourier transform of 05

$$f(x) = a^2 - x^2, if\ |x| < a$$
$$= 0, if\ |x| > a$$