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SUBJECT CODE-11
FACULTY OF ENGINEERING AND TECHNOLOGY
S.E. (All Branches) Examination Nov/Dec 2015
Engineering Mathematics -IV
(Revised)

[Time: Three Hours] **[Max. Marks: 80]**

"Please check whether you have got the right question paper."

N.B i) Question Numbers 1 and 6 are compulsory
ii) Solve any two questions from remaining of each section.
iii)figures to the right indicate full marks
iv) Assume suitable data , if necessary

Section-A

Q1. Solve any five:- 10

a) Verify the Cauchy Reimann equations for the function W=log Z

b) Verify the Laplace equation for u=(r+$\frac{a^2}{r}$) cos θ

c) Evaluate $\int_c e^z \, dz$, where c is |z|=1

d) Evaluate $\int_{(0,0)}^{(1=i)} (z^2 + z) \, dz$, along y=$x^2$

e) Find the residue of f (z)=$\frac{1}{(z^2+a^2)^2}$ at z= ai

f) Find the image of the line x=0 under the transformation w=e^z

g) Solve the equation$\frac{\partial^2 z}{\partial y^2} - z = 0$, $when\ y = 0, z = e^x\ and\ \frac{\partial z}{\partial y} = e^{-x}$

OR

Find the z-Transform of 2^k, K< 0

h) Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, Where u (0,y)= 8 e^{-3y}

OR

Find the z-transform of cos 2 K

Q.2 a) Find the analytic function whose imaginary part is $2e^{-y}$(y cos x+ x sin x) 05

b) Evaluate $\int_c \frac{Z}{(z^2-6z+25)^2}$ dz, where C is 1z-3-4 i l=4, by using cauchy's integral formula 05

c) Solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, subject to the conditions 05

$$y(0,t) = y(l,t) = 0, \left(\frac{\partial y}{\partial t}\right)_{t=0} = 0 \; for \; 0 \le x \le l \; and \; y\,(x,0) = \frac{4\lambda\, x\,(l-x)}{l^2}$$

OR

Find the z- transform of $\cos^2 \frac{k\pi}{6}$

Q.3 a) Find the harmonic conjugate of $u = e^{a\,(x^2-y^2)}$Cos 2 xy. Also find corresponding analytic function f(z) 05

b) Evaluate $\int_c |z|\bar{z}dz$, where c is the closed curve consisting of the upper semicircle of |z|=1 and the segment -1$\le x \le 1$ 05

c) Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $for\ 0 < x < \pi$, u_x(0,t)= 0 05

$u_{x\,(\pi,t)}$=0 $and\ u\,(x,0)$=$\sin x$.

OR

Find the inverse z- transform of $\frac{z^2}{z^2+1}$ 05

Q.4 a) Find and plot the rectangular region 0$\le x \le 1, 0 \le y \le 2$ under the transformation 05

W= $\sqrt{2}\, e^{\frac{i\pi}{4}} z + (1-2i)$

b) Evaluate $\oint_c \frac{ze^z}{(z^2+9)}$dz, where c is |z|=5, by cauchy's Residue Theorem 05

c) Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, for\ 0 < x < \pi,$ 05

0<Y<π, given that u(0,y)= u $(\pi, y) = u\,(x, \pi) = 0$ and $u(x, 0) = x$

OR

Solve y (k+2)-5y(k+1)+6y(k)=u(k),y(0)=o, y(1)=1, by z-transform 05

Q.5 a) Find the bilinear transformation which maps the points z=1,i, -1 on to W=2,i,-2 05

b) Expand f(z)=$\frac{z}{(z-1)(z-3)}$, for 1<|z-1|<$\frac{3}{2}$ 05

c) Evaluate $\int_0^{2\pi} \frac{d\theta}{(13+5sin\theta)}$ by Residue Theorem 05

Section -B

Q.6 Solve any five: 10

a) Find the Laplace transform of $\frac{Sin\ 2t}{t}$

b) Find the Laplace transform of $e^{-t}t^{3/2}$

c) Find the Laplace transform of [sin 2t- cos 2t]^2

d) Find the Laplace transform of $\frac{1}{s}\left(\frac{s-2}{s+2}\right)$

e) Find the inverse Laplace transform of $\frac{s+1}{s^2-6s+25}$

f) Find the inverse Laplace transform of $\frac{e^{-s}}{s\,(s^2+4)}$

g) Find f(x), if Fourier sin e transform is $\frac{\pi}{2}$

h) Find the Fourier transform of

F(x)=$\frac{1}{2a}, if\ |x| \leq a$

= 0, if $|x| > a$

Q.7 a) Evaluate $\int_0^\infty e^{\sqrt{2}t}\ \frac{\sin\sqrt{2}\,t}{t}$ dt 05

b) Find the inverse Laplace transform of $\frac{1}{s}\log\left(1-\frac{a^2}{S^2}\right)$ 05

c) Solve $\frac{\partial u}{\partial t}$= 2 $\frac{\partial^2 u}{\partial x^2}$, subject to the conditions 05

i. U (0,t)=0
ii. U(x,0)=$\bar{e}^x\ and$
iii. U(x, t) is bounded

Q.8 a) Find the Laplace transform of $e^{3t}\ \int_0^t t\ sinh2t\ dt$ 05

b) Find the inverse Laplace transform of $\frac{s+3}{(s^2+6s+13)^2}$ by convolution theorem 05

c) Find the Fourier cosine transform of $\frac{e^{-ax}-e^{-bx}}{x}$ 05

Q.9 a) Find the Laplace transform of periodic function 05

f(t)=t, o < t < c

=2(-t , c< t < 2 c and f (t)= f(t+2c)

b) Solve $\frac{dy}{dt} + 3y = 10 \sin t, y(0) = 0$ by laplace transform method. 05

c) Solve the integral equation $\int_0^\infty f(x) \cos pxdx = 1 - p,\ 0 < p < 1$ 05

$= 0,\ p > 1$

Q.10 a) Express the following function in terms of Heaviside unit step function and hence find their Laplace transform, f(t)=$e^{-t}, 0 < t < 3$ 05

= 0, t > 3

b) Solve $\frac{dx}{dt}$+ 4y = 0, $\frac{dy}{dt}$ -9x = 0, x(0)=2,y(0)=1 by Laplace transform 05

c) Find the Fourier transform of 05

F(x)=Sin x, if 0<x<1

= 0, otherwise