Total No. of Printed Pages:3

SUBJECT CODE NO:- P-220
FACULTY OF ENGINEERING AND TECHNOLOGY
S.E. (All Branches) Examination May/June 2017
Engineering Mathematics -IV
(Revised)

[Time: Three Hours] **[Max.Marks:80]**

Please check whether you have got the right question paper.

N.B
i) Q.No.1 from and Q.No.6 are compulsory.
ii) Solve any two questions from remaining of each section.
iii) Figures to the right indicate full marks.
iv) Assume suitable data, if necessary.

Section A

Q.1 Solve any five from the following: 10

a. Find the analytic function whose imaginary part is e^xsiny.

b. Show that u= $\bar{e}^{\theta}$ cos (log r) is harmonic.

c. Find the image of the line y=2x, under the transformation W=Z^2

d. Evaluate $\int_0^{1+i}(x^2-iy)dz$ along the line y=x.

e. Evaluate $\int_c \frac{e^z}{z}$ dz, where c is $|z|$=1

f. Find the poles of the function and the corresponding residues at each pole of f(z)= $\frac{ze^z}{(z+1)^3}$

g. Solve $\frac{\partial u}{\partial x}$=$\frac{2\partial u}{\partial t}$+u, where u(x, 0)=6$e^{-3x}$.

OR

Find the Z-transform of f(k) =k, k$\geq o$.

h. Solve $\frac{\partial^2 z}{\partial x \partial y}$=sinx.

OR

Find the Z-transform of e^{-ak},k≥0,

Q.2 a. Show that the function f(z)=e^x(cosy + i sin y) is analytic and find its derivative. 05

b. Find the bilinear transformation which maps the point z=-1, 0, 1 onto the points W=0,i,3i. 05

c. Find the Z-transform of $\frac{\cos 2k}{k}$, k≥o. 05

OR

Solve $\frac{\partial^2 y}{\partial t^2}$=$C^2\frac{\partial^2 y}{\partial x^2}$, subject to the conditions 05
Y(0,t)=0, Y(l,t)=0, $\partial y/\partial t$ =0 at t=0
And y(x,0)=$\frac{3a}{2l}$x, 0≤ $x \leq \frac{2l}{3}$
=$\frac{3a}{l}$(l-x), $\frac{2l}{3} \leq x \leq l.$

Q.3 a. Find k such that f(x, y)=x^3+3kxy^2 may be harmonic and find its conjugate harmonic function. 05

b. Evaluate $\int_c \bar{z}^2 dz$, Where c is $|Z-1|$=1. 05

c. Find the inverse Z-transform of $\frac{Z}{(z-2)(z-3)}$,$|Z|>3.$ 05

OR

Solve $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ for $o< x<\pi, t>o$ 05

$\frac{\partial u}{\partial x}$ =0 at x=0, $\frac{\partial u}{\partial x}$=0 at x= π and u(x, o) =sinx.

Q.4 a. Expand f(z)=$\frac{1}{(z+1)(z+2)}$ for $0<|Z-1|<1$. 05

b. Evaluate $\oint_C \frac{sinz}{(z-1)^2(z^2-9)}$ dz, where c is $|Z-3|=\frac{1}{2}$. By cauchy's integral formula. 05

c. Solve the difference equation by Z-transform $u_{K+2}-2u_{k+1}+u_k=2^K$, with $Y_o=2$, $Y_1=1$. 05

OR

Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, subject to the conditions 05

u(0, y) = u(π, y)=0 for all y $\geq$ 0 and u(x, 0)=100 u(x, ∞)=0.

Q.5 a. Under the transformation W=Z+$\frac{a^2}{z}$, show that the map of the circle $x^2+y^2=a^2$ is a straight line, but the map of the circle $x^2+y^2=b^2$ (b>a) is an ellipse. 05

b. Evaluate $\oint_C \frac{z^2}{\sin^3 z\, cosz}$ dz, where c is $|Z+i|$=2 by cauchy's Residue theorem. 05

c. Evaluate $\int_{-\pi}^{\pi}\frac{1}{1+\sin^2\theta}d\theta$, by using Residue theorem. 05

Section-B

Q.6 Solve any five from the following: 10

a. Find Laplace transform of $te^{-2t}\delta(t-2)$.

b. Find L[$f(t)$] and L[$f'(t)$] of the following function f(t)=3, 0≤ $t<5$

=0, t>5.

c. Find the Laplace transform of f(t)= $(t-2)^2$,t>2

=0, t<2

d. Find inverse Laplace transform of $\frac{2s+2}{s^2+2s+10}$

e. Find inverse Laplace transform of $\frac{e^{-\pi s}}{s^2+9}$

f. find inverse Laplace transform of $s^{\frac{-7}{2}}$

g. Find the Fourier cosine transform of f(x)=k, 0<x<a

=0,x>a

h. Find the Fourier transform of f(x)= x, 0<x<a

=0, other wise

Q.7 a. Find the Laplace transform of $\int_o^t \frac{1+e^t}{t}$dt. 05

b. Find the inverse Laplace transform of $\tan^{-1}\frac{2}{5}$ 05

c. Using Fourier transform , solve the equation $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$, 0<x<∞, t>0 05

Subject to the conditions

u(0,t)=0, t>0, u(x,0)=e^{-x}, x>0,

u and $\frac{\partial u}{\partial x}$ →0 as x→ ∞.

Q.8 A. Evaluate $\int_0^\infty e^{-5t}\sinh^3 t\, dt$ 05

b. Find the inverse Laplace transform by convolution theorem of $\frac{1}{s(s^2+4)}$ 05

c. Find f(x) satisfying the integral equation $\int_0^\infty f(x)sin\lambda x dx=\frac{sin\lambda}{\lambda}$ 05

Q.9 Express the following function in terms of Heaviside unit step function and hence find their Laplace transform 05

F(x)=sin t, 0<t<π

=t, t>π

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b. Solve $y'' - 6y' + 9y = t^2 e^{3t}$, y(0)=2, y' (0)=6 by Laplace transform. 05

c. Find the Fourier sine transform of 05

f(x)=x, 0<x<1

=2-x, 1<x<2

=0, x>2

Q.10 a. Find the Laplace transform of f(t)= e^t, $0<t<2\pi$, $f(t)=f(t+2\pi)$. 05

b. Solve $\frac{dx}{dt}$+y=sin t , $\frac{dy}{dt}$+x=cost, X(0)=2, y(0)=0 by Laplace transform. 05

c. Find the Fourier transform of f(x)=$\frac{1}{2a}$, if $|x| \leq a$ 05

=0, if $|x|>a$.